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EXPERIMENTAL ANALYSIS OF CLASSICAL LUMPED MODEL FOR TRANSIENT HEAT CONDUCTION IN SLAB

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ABSTRACT

The present work aims at applying the ideas on the analysis of improved lumped-parameter model for transient heat conduction in a slab with temperature-dependent thermal conductivity. The transient temperature is found to depend on various model parameters, namely, Biot number, heat source parameter and time. Polynomial Approximation Method (PAM) has been possible to derive a unified relation for the transient thermal behavior of solid (slab and tube) with both internal generation and boundary heat flux. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time.

An improved lumped parameter model has been received through two point Hermite approximations for integrals. For directly temperature-subordinate warm conductivity, it is demonstrated by correlation with numerical arrangement of the first dispersed parameter model that the higher request lumped model (H1,1/H0,0 approximation) yields critical change of normal temperature expectation over the established lumped model. A brought together Biot number cutoff relying upon a solitary dimensionless parameter □ is given both for cooling and warming methods. The aftereffect of the present investigation has been contrasted and before numerical and logical results. A decent understanding has been gotten between the present forecast and the accessible result.

Key words: Hermite approximations, PAM, Temperature-dependent thermal conductivity, Lumped model, Nonlinear heat Conduction, Transient heat conduction, Biot number.

INTRODUCTION

In this chapter introduces the mechanism of heat transfer known as conduction. In the connection of building applications, this is more inclined to be illustrative of the conduct in strong than liquids. Conduction phenomena may be dealt with as either time-indigent or unfaltering satisfy. Time-subordinate conduction has been improved to the amazing instances of Bi << 1 and Bi >> 1. For the previous, the lumped method may be utilized and as a part of the recent the semi-infinite method. It is important that in both cases these method are utilized as a part of functional applications in the opposite mode to gauge warmth exchange coefficient from a known temperature-time history.

METHODOLOGY

According to Lumped Body models we first introduce the spatially averaged dimensionless temperature as follows:

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial\eta}\left(\lambda(\theta)\frac{\partial\theta}{\partial\lambda}\right),\quad \text{in } 0<\lambda<1 \text{ for } \tau>0, \quad (1)$$

$$\theta_{av}(\tau)=\int_0^1\theta(\eta,\tau)\,d\eta \qquad (2)$$

We operate Eq. (1) by $\int_0^1 d\eta$, using the definition of average temperature, Eq. (2), we get

$$\frac{d\theta_{av}(\tau)}{\partial\tau}=\left(\frac{\lambda(\theta)\partial\theta}{\partial\eta}\right)_{\eta=1}-\left(\frac{\lambda(\theta)\partial\theta}{\partial\eta}\right)_{\eta=0} \qquad (3)$$

Now, the boundary conditions

$$\frac{d\theta_{av}(\tau)}{\partial\tau}=-B_i\theta(1,\tau) \qquad (4)$$

Eq. (4) is an equivalent integral-differential formulation of the mathematical model with no approximation involved.

Assuming that the temperature angle is adequately uniform over the entire spatial arrangement area, the traditional lumped framework examination (CLSA) is in light of the suspicion that the limit temperatures can be sensibly very much approximated by the normal temperature, as

$$\theta(0,\tau)\cong\theta(1,\tau)\cong\ \theta_{av}(\tau),$$

Which leads to the classical lumped model

$$\frac{d\theta_{av}(\tau)}{\partial\tau}=-B_i\theta_{av}(\tau)$$

And to be solved with the initial condition for the average temperature

$$\theta_{av}(0)=1$$

It can be seen that the classical model shows no influence of the temperature-dependent thermal conductivity.

Alhama and Zueco identified four different kinds of problem that may occur: (i) a heating process with a positive temperature-dependent coefficient, $k_2 > 0$; (ii) a heating process with $k_2 < 0$; (iii) a cooling process with $k_2 > 0$ and (iv) a cooling process with $k_2 < 0$. They established that the universal mean Biot number limit for applying the lumped model can be expressed as a function of the dimensionless number k = (k_{max} - k_{min})/k_m, and the kind of process (cooling or heating), with k_m = (k_{max} - k_{min})/2

In proper choice of dimensionless parameters, the four kinds of problem can be reduced to two kinds of problem: (i) β > 0, representing cooling with a positive temperature-dependent coefficient b > 0 or heating with b < 0 and (ii) β < 0, representing cooling with b < 0 or heating with b > 0. The main difference between Alhama–Zueco's analysis and ours lies in the choice of the reference temperature. While Alhama and Zueco always use the minimum temperature T_{min} as the reference temperature, we always use the surrounding fluid temperature T_∞ as the reference temperature whether cooling or heating. For a linearly temperature dependent thermal conductivity.

$$k(T) = k_\infty\{1 + b(T - T_\infty)\}$$

We have for a cooling process ($T_i > T_\infty$) with a positive temperature dependent coefficient
(b > 0)

$$\lambda(\theta) = \frac{k(T)}{k_\infty} = 1 + b(T_i - T_\infty)\theta = 1 + \beta\theta$$

Thus $\beta = b(T_i - T_\infty) > 0$. For a cooling process with b < 0, we have

$$\lambda(\theta) = 1 + b(T_i - T_\infty)\theta = 1 + \beta\theta$$

with $\beta = b(T_i - T_\infty) < 0$. For a heating process $(T_i < T_\infty)$ with b > 0, we have

$$\lambda(\theta) = 1 + b(T_i - T_\infty)\theta = 1 + \beta\theta$$

with $\beta = b(T_i - T_\infty) > 0$.

It can be seen that the four kinds of problem identified by Alhama and Zueco [2] can be represented conveniently by only one dimensionless parameter β, with $\beta > 0$ representing cooling with b > 0 or heating with b < 0, and $\beta < 0$ representing cooling with b < 0 or heating with b > 0. We proceed to examine the example problems given by Alhama and Zueco.

Problem 1. $T_i = 1, T_\infty = 0$, k(T) = 0.9+0.2T, k_{min} = 0.9 , k_{max} = 1.1 , k_m = 1, k = 0.2 By our analysis, θ = T, $\theta_i = 1$, $\theta_\infty = 0$,

$\lambda(\theta) = kT/k_\infty = 1+(2/9)\theta$, β = 2/9

Problem 2. $T_i = 1, T_\infty = 0$, k(T) = 1.8+0.4T, k_{min} = 1.8 , k_{max} = 2.2 , k_m = 2, k = 0.2 By our analysis, θ = T, $\theta_i = 1$, $\theta_\infty = 0$, $\lambda(\theta) = kT/k_\infty = 1+(2/9)\theta$, β = 2/9

Problem 3. $T_i = 10, T_\infty = 0$, k(T) = 0.9+0.02T, k_{min} = 0.9, k_{max} = 1.1 , k_m = 1, k = 0.2 By our analysis, θ = T/10, $\theta_i = 1$, $\theta_\infty = 0$,

$\lambda(\theta) = kT/k_\infty = 1+(2/9)\theta$, β = 2/9

The difference between Alhama-Zueco′s and our analysis is shown when examining the heating processes with a positive temperature-dependent coefficient ($k_2 > 0$ or b > 0).

Problem 4. $T_i = 0, T_\infty = 1$, k(T) = 0.9+0.2T, k_{min} = 0.9 , k_{max} = 1.1 , k_m = 1, k = 0.2 By our analysis, θ = (T – 1)/(-1), $\theta_i = 1$, $\theta_\infty = 0$,

$$\lambda(\theta) = \frac{kT}{k_\infty} = \frac{0.9+0.2(-\theta+1)}{1.1} = 1 - \frac{2}{11}\theta,$$

Thus β = 2/11

Problem 5. $T_i = 0, T_\infty = 1$, k(T) = 1.8+0.4T, k_{min} = 1.8 , k_{max} = 2.2 , k_m = 2, k = 0.2 By our analysis, θ = (T – 1)/(-1), $\theta_i = 1$, $\theta_\infty = 0$,

$$\lambda(\theta) = \frac{kT}{k_\infty} = \frac{1.8+0.4(-\theta+1)}{2.2} = 1 - \frac{2}{11}\theta,$$

Thus β = 2/11

Problem 6. $T_i = 0, T_\infty = 10$, k(T) = 0.9+0.02T, k_{min} = 0.9, k_{max} = 1.1, k_m = 1,k = 0.2 By our analysis, θ = (T – 10)/(-10), $\theta_i = 1$, $\theta_\infty = 0$,

$$\lambda(\theta) = \frac{kT}{k_\infty} = \frac{0.9+0.02\times(-\theta+10)}{2.2} = 1 - \frac{2}{11}\theta,$$

Thus β = 2/11

It can be seen that problems 1-3 reduce to a same dimensionless problem with β = 2/9 and problem 4-6 reduce to another dimensionless problem with β = -2/11

RESULT AND DISCUSSION

The arrangements of established and enhanced lumped models are indicated in even and graphical structures in correlation with a reference limited contrast arrangement of the first dispersed model, The starting limit esteem issue characterized by utilizing a certain limited distinction method, with a 201 hubs work in spatial discretization and a dimensionless time venture of 0.00001 for all cases. Diverse estimations of the Biot number Bi and the parameter b are picked in order to evaluate precision of the arrangements given by lumped models. In Table 4.1, it is presented a comparison of the dimensionless average temperatures obtained by lumped models and the reference finite difference solution of the original distributed parameter model at different values of time, for B_i = 1.0 and β =1.0. As can be seen, the classical lumped model gives an error of 0.0681 at τ = 1.0, while the $H_{0,0}/H_{0,0}$ model gives an error of 0.0137 at τ = 1.0, and the $H_{1,1}/H_{0.0}$ model yields a maximum error less than 0.005 for all time values. Fig.4.1 shows the comparison of the dimensionless average temperatures for B_i =2.5 and β= 0.5. It can be seen that the solution give by the higher order improved lumped model ($H_{1,1}/H_{0,0}$) agrees quite well with the finite difference solution.

Table 1
Comparison of lumped model against finite different solution average temperature $\theta_{av}(\tau)$ at different value of time

τ	FD solution $B_i = 1.0$	CLSA $\beta = 1.0$	$H_{0,0}/H_{0,0}$	$H_{1,1}/H_{0,0}$
0.1	0.9150	0.9048	0.9157	0.9190
0.2	0.8406	0.8187	0.8389	0.8450
0.3	0.7730	0.7408	0.7689	0.7774
0.4	0.7113	0.6703	0.7050	0.7156
0.5	0.6548	0.6065	0.6466	0.6589
0.6	0.6031	0.5488	0.5934	0.6070
0.7	0.5557	0.4966	0.5447	0.5595
0.8	0.5123	0.4493	0.5002	0.5159
0.9	0.4725	0.4066	0.4595	0.4758
1.0	0.4359	0.3679	0.4222	0.4391
2.0	0.1985	0.1353	0.1838	0.1997
3.0	0.0925	0.0498	0.0813	0.0926
4.0	0.0436	0.0183	0.0363	0.0434
5.0	0.0207	0.0067	0.0163	0.0204

CONCLUSION

The improved lumped parameter models are presented for transient heat conduction in a slab with cubicly temperature-dependent thermal conductivity and subject to convective cooling or heating. Improved lumped models are obtained through two point Hermite approximations for integrals. For linearly temperature-dependent thermal conductivity, it is shown by comparison with numerical solution of the original distributed parameter model that the higher order lumped model ($H_{1,1}/H_{0,0}$ approximation) yields significant improvement of average temperature prediction over the classical lumped model. It is shown that the maximum relative error of the dimensionless average temperature is influenced predominantly by the Biot number. A unified Biot number limit is obtained as a function of the linear dependence coefficient β, $_{\text{Bilimit}} = 0.523\beta + 1.078$ for $-0.6 \leq \beta \leq 0.8$. The lumped model $H_{1,1}/H_{0,0}$ is expected to yield maximum normalized error less 0.01 for $B_i < B_{ilimit}$, for a given β.

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